Module 2

IIR filter design

Using Impulse Invarient Transformation(IIT)

Preliminaries:-IIR filter design

- IIR filters have infinite-length impulse responses, hence they can be matched to analog filters.
- Analog filter design is a mature and well developed field.

Preliminaries:-IIR filter design

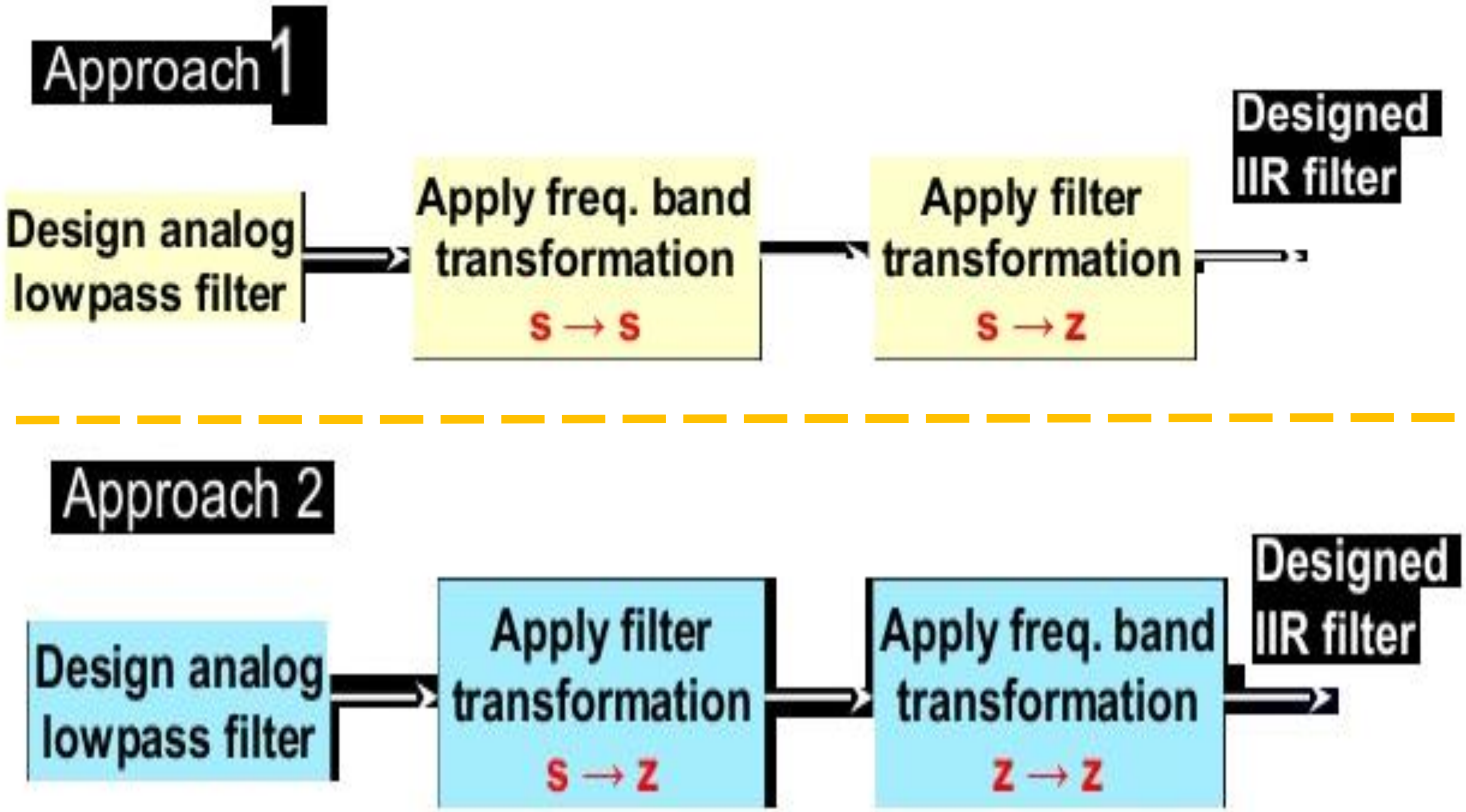
- First design filter in the analog domain
- Then convert the design into the digital domain.
- Using analog to digital transformation techniques.

What are the methods to convert analog filter into digital filter?

- There are many techniques which are **used to convert analog filter into digital filter** of which some of them are:-

1) Approximation of derivatives
2) Bilinear Transformation(BLT)
3) Impulse invariance Transformation(**IIT**)
4) Matched Z-transformation

Filter design approaches



Analog to Digital filter Transformation:-

Method 1 (IIT)

Impulse invariance Transformation(IIT)

Analog to Digital filter Transformation:-

Method 1 (IIT)

IIT Derivation for simple pole Transformation

- Our aim is to transform filter from s to z domain
- -consider the simplest case of H(s) with simple pole

$$H(s) = \sum_{i=1}^{N} \frac{A_i}{s + p_i} = \frac{A_1}{s + p_1} + \frac{A_2}{s + p_2} + \ldots + \frac{A_N}{s + p_N}$$

Analog to Digital filter Transformation:-

Method 1 (IIT) Derivation

$$H(s) = \sum_{i=1}^{N} \frac{A_i}{s + p_i} = \frac{A_1}{s + p_1} + \frac{A_2}{s + p_2} + \ldots\ldots + \frac{A_N}{s + p_N}$$

Let, $h(t)$ = Impulse response of analog filter

The Laplace transform of the analog impulse response $h(t)$ gives the transfer function of analog filter.

$\therefore$ Transfer function of analog filter, $H(s) = \mathcal{L}\{h(t)\}$.

When $H(s)$ has N number of distinct poles, it can be expressed as shown in equation (7.1) by partial fraction expansion.

Analog to Digital filter Transformation:-

Method 1 (IIT) Derivation….contd..

When H(s) has N number of distinct poles, it can be expressed as shown in equation (7.1) by partial fraction expansion.

$$H(s) = \sum_{i=1}^{N} \frac{A_i}{s + p_i} = \frac{A_1}{s + p_1} + \frac{A_2}{s + p_2} + \ldots\ldots + \frac{A_N}{s + p_N} \quad \ldots\ldots(7.1)$$

$$\mathcal{L}\{e^{-at}\,u(t)\} = \frac{1}{s+a}$$

On taking inverse Laplace transform of equation (7.1) we get,

$$h(t) = \sum_{i=1}^{N} A_i\, e^{-p_i t}\, u(t) = A_1\, e^{-p_1 t}\, u(t) + A_2\, e^{-p_2 t}\, u(t) + \ldots\ldots + A_N\, e^{-p_N t}\, u(t) \quad \ldots(7.2)$$

where, u(t) = Continuous time unit step function.

Let, T = Sampling period.

Analog to Digital filter Transformation:-

Method 1 (IIT) Derivation....contd..

Let, T = Sampling period.

$h(n)$ = Impulse response of digital filter.

The impulse response of the digital filter is obtained by uniformly sampling the impulse response of the analog filter.

$$\therefore h(n) = h(t)\Big|_{t=nT} = h(nT)$$

Therefore the impulse response h(n) can be obtained from equation (7.2) by replacing t by nT.

$$\therefore h(n) = h(t)\Big|_{t=nT} = h(nT) = \sum_{i=1}^{N} A_i\, e^{-p_i nT}\, u(nT)$$

$$= A_1 e^{-p_1 nT} u(nT) + A_2 e^{-p_2 nT} u(nT) + \ldots + A_N e^{-p_N nT} u(nT) \qquad .....(7.3)$$

Analog to Digital filter Transformation:-

Method 1 (IIT) Derivation….contd..

$$\therefore\ h(n) = h(t)\Big|_{t=nT} = h(nT) = \sum_{i=1}^{N} A_i\, e^{-p_i nT}\, u(nT)$$

$$= A_1 e^{-p_1 nT} u(nT) + A_2 e^{-p_2 nT} u(nT) + \ldots + A_N e^{-p_N nT} u(nT) \qquad \ldots(7.3)$$

On taking $\mathcal{Z}$-transform of equation (7.3) we get,

$$\boxed{\mathcal{Z}\left\{e^{-anT}\, u(nT)\right\} = \frac{1}{1-e^{-aT} z^{-1}}}$$

$$H(z) = \mathcal{Z}\{h(n)\} = A_1 \frac{1}{1 - e^{-p_1 T} z^{-1}} + A_2 \frac{1}{1 - e^{-p_2 T} z^{-1}} + \ldots$$

$$+ A_N \frac{1}{1 - e^{-p_N T} z^{-1}} = \sum_{i=1}^{N} A_i \frac{1}{1 - e^{-p_i T} z^{-1}} \qquad \ldots(7.4)$$

Analog to Digital filter Transformation:-

Method 1 (IIT) Derivation….contd..

$$H(s) = \sum_{i=1}^{N} \frac{A_i}{s + p_i} = \frac{A_1}{s + p_1} + \frac{A_2}{s + p_2} + \cdots + \frac{A_N}{s + p_N}$$

$$H(z) = Z\{h(n)\} = \quad + A_N \frac{1}{1 - e^{-p_N T} z^{-1}} = \sum_{i=1}^{N} A_i \frac{1}{1 - e^{-p_i T} z^{-1}}$$

$$\frac{1}{s + p_i} \xrightarrow{\text{(is transformed to)}} \frac{1}{1 - e^{-p_i T} z^{-1}}$$

Analog to Digital filter Transformation:-

Method 1 (IIT) ..Example

$$\frac{1}{s+p_i} \xrightarrow{\text{(is transformed to)}} \frac{1}{1-e^{-p_i T} z^{-1}}$$

- Use of above formula for transforming following analog filter to digital filter

$$H(s) = \frac{2}{s^2+3s+2} = \frac{2}{(s+1)(s+2)}$$

Analog to Digital filter Transformation:-

AJN notes Method 1 (IIT) ..Example

$$H(s) = \frac{2}{s^2 + 3s + 2} = \frac{2}{(s+1)(s+2)}$$

$$H(s) = \frac{2}{s+1} + \frac{-2}{s+2}$$

By impulse invariant transformation we know that,

$$\frac{A_i}{s + p_i} \xrightarrow{\text{(is transformed to)}} \frac{A_i}{1 - e^{-p_i T} z^{-1}}$$

$$\therefore\ H(z) = \frac{2}{1 - e^{-p_1 T} z^{-1}} + \frac{-2}{1 - e^{-p_2 T} z^{-1}} \quad \text{where } p_1 = 1 \text{ and } p_2 = 2$$

$$H(z) = \frac{2}{1 - e^{-T} z^{-1}} + \frac{-2}{1 - e^{-2T} z^{-1}}$$

$$\frac{0.4652 z^{-1}}{1 - 0.5032 z^{-1} + 0.0498 z^{-2}}$$

O

Analog to Digital filter Transformation:-Method 1 (IIT) Important formula for different types of poles

$$\frac{1}{s-p_i} \longrightarrow \frac{1}{1-e^{p_i T} z^{-1}}$$

2. For multiples poles

$$\frac{1}{(s+p_i)^m} \longrightarrow \frac{(-1)^{m-1}}{(m-1)!} \frac{d^{m-1}}{dp_i^{m-1}} \left(\frac{1}{1-e^{-p_i T} z^{-1}} \right)$$

3. For complex poles

$$\frac{s+a}{(s+a)^2+b^2} \longrightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2} \longrightarrow \frac{1-e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

For the analog transfer function, $H(s) = \frac{2}{s^2 + 3s + 2}$, determine H(z) using impulse invar
transformation if (a) T = 1 second and (b) T = 0.1 second.

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

Solution

Given that, $H(s) = \dfrac{2}{s^2 + 3s + 2}$

- **Step 1:**-calculate partial fractions to get H(S) in one of the standard formula of IIT

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

AJN notes

$$\frac{1}{s-p_i} \longrightarrow \frac{1}{1-e^{p_i T}z^{-1}}$$

2. For multiples poles

$$\frac{1}{(s+p_i)^m} \longrightarrow \frac{(-1)^{m-1}}{(m-1)!}\frac{d^{m-1}}{dp_i^{m-1}}\left(\frac{1}{1-e^{-p_i T}z^{-1}}\right)$$

3. For complex poles

$$\frac{s+a}{(s+a)^2+b^2} \longrightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2} \longrightarrow \frac{1-e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

Solution

Given that, $H(s) = \frac{2}{s^2 + 3s + 2} = \frac{2}{(s+1)(s+2)}$

By partial fraction expansion technique we can write,

$$H(s) = \frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

Solution

Given that, $H(s) = \dfrac{2}{s^2 + 3s + 2} = \dfrac{2}{(s+1)(s+2)}$

By partial fraction expansion technique we can write,

$$H(s) = \frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \frac{2}{\cancel{(s+1)}(s+2)} \times \cancel{(s+1)}\Bigg|_{s=-1} = \frac{2}{-1+2} = 2$$

$$B = \frac{2}{(s+1)\cancel{(s+2)}} \times \cancel{(s+2)}\Bigg|_{s=-2} = \frac{2}{-2+1} = -2$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

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$$\therefore H(s) = \frac{2}{s+1} + \frac{-2}{s+2}$$

By impulse invariant transformation we know that,

$$\frac{A_i}{s+p_i} \xrightarrow{\text{(is transformed to)}} \frac{A_i}{1-e^{-p_iT} z^{-1}}$$

$$\therefore H(z) = \frac{2}{1-e^{-p_1T} z^{-1}} + \frac{-2}{1-e^{-p_2T} z^{-1}} \quad \text{where } p_1 = 1 \text{ and } p_2 = 2$$

$$H(z) = \frac{2}{1-e^{-T} z^{-1}} + \frac{-2}{1-e^{-2T} z^{-1}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

$$H(z) = \frac{2}{1-e^{-1}z^{-1}} + \frac{-2}{1-e^{-2}z^{-1}}$$

$$H(z) = \frac{2}{1-0.3679z^{-1}} + \frac{-2}{1-0.1353z^{-1}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

$$H(z) = \frac{2}{1 - e^{-1} z^{-1}} + \frac{-2}{1 - e^{-2} z^{-1}}$$

$$H(z) = \frac{2}{1 - 0.3679 z^{-1}} + \frac{-2}{1 - 0.1353 z^{-1}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

AJN notes

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$$H(z)=\frac{2}{1-e^{-1}z^{-1}}+\frac{-2}{1-e^{-2}z^{-1}}$$

$$H(z)=\frac{2}{1-0.3679z^{-1}}+\frac{-2}{1-0.1353z^{-1}}=\frac{2(1-0.1353z^{-1})-2(1-0.3679z^{-1})}{(1-0.3679z^{-1})(1-0.1353z^{-1})}$$

$$=\frac{2-0.2706z^{-1}-2+0.7358z^{-1}}{1-0.1353z^{-1}-0.3679z^{-1}+0.0498z^{-2}}=\frac{0.4652z^{-1}}{1-0.5032z^{-1}+0.0498z^{-2}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1..part 2 ..T=0.1

Solve for Z to power -1 coefficients

$$H(z) = \frac{2}{1 - e^{-0.1} z^{-1}} + \frac{-2}{1 - e^{-0.2} z^{-1}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1..part 2 ..T=0.1

Solve for Z to power -1 coefficients

$$H(z) = \frac{2}{1 - e^{-0.1} z^{-1}} + \frac{-2}{1 - e^{-0.2} z^{-1}}$$

$$= \frac{2}{1 - 0.9048 z^{-1}} + \frac{-2}{1 - 0.8187 z^{-1}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

AJN notes

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$$H(z)=\frac{2}{1-e^{-0.1}z^{-1}}+\frac{-2}{1-e^{-0.2}z^{-1}}$$

$$=\frac{2}{1-0.9048z^{-1}}+\frac{-2}{1-0.8187z^{-1}}=\frac{2(1-0.8187z^{-1})-2(1-0.9048z^{-1})}{(1-0.9048z^{-1})(1-0.8187z^{-1})}$$

$$=\frac{2-1.6374z^{-1}-2+1.8096z^{-1}}{1-0.8187z^{-1}-0.9048z^{-1}+0.7408z^{-2}}=\frac{0.1722z^{-1}}{1-1.7235z^{-1}+0.7408z^{-2}}$$

O

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

Convert the analog filter with system transfer function,

$$H(s) = \frac{(s+0.1)}{(s+0.1)^2+9}$$

into a digital IIR filter by means of the impulse invariant method.

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

Given that, $H(s) = \frac{(s+0.1)}{(s+0.1)^2+9} = \frac{s+0.1}{s^2+0.2s+9.01}$

The roots of the quadratic $s^2 + 0.2s + 9.01 = 0$ are

$$s = \frac{-0.2 \pm \sqrt{0.2^2 - 4 \times 9.01}}{2}$$

$$= \frac{-0.2}{2} \pm \frac{1}{2}\sqrt{-36} = -0.1 \pm j3$$

$$\therefore \ (s^2 + 0.2s + 9.01)$$

$$= (s - (-0.1 + j3))(s - (-0.1 - j3))$$

$$= (s + 0.1 - j3)(s + 0.1 + j3)$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

Method - II

Given that, $H(s) = \dfrac{(s+0.1)}{(s+0.1)^2+9} = \dfrac{s+0.1}{s^2+2\times 0.1\times s+0.1^2+9}$

$$= \frac{s+0.1}{s^2+0.2s+9.01} = \frac{s+0.1}{(s+0.1-j3)\,(s+0.1+j3)}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

By partial fraction expansion H(s) can be expressed as,

$$H(s) = \frac{s+0.1}{(s+0.1-j3)(s+0.1+j3)} = \frac{A}{s+0.1-j3} + \frac{A^*}{s+0.1+j3}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

AJN notes

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$$H(s)=\frac{s+0.1}{(s+0.1-j3)(s+0.1+j3)}=\frac{A}{s+0.1-j3}+\frac{A^*}{s+0.1+j3}$$

$$A=\frac{s+0.1}{\cancel{(s+0.1-j3)}(s+0.1+j3)}\times\cancel{(s+0.1-j3)}\Bigg|_{s=-0.1+j3}=0.5$$

$$A^*=(0.5)^*=0.5$$

$$\therefore\ H(s)=\frac{0.5}{s+0.1-j3}+\frac{0.5}{s+0.1+j3}$$

O

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

$$\therefore \; H(s) = \frac{0.5}{s + 0.1 - j3} + \frac{0.5}{s + 0.1 + j3}$$

By impulse invariant transformation we know that,

$$\frac{A_i}{s + p_i} \xrightarrow{\text{(is transformed to}} \frac{A_i}{1 - e^{-p_i T} z^{-1}} \quad \text{and let,} \quad T = 1$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

$$\therefore\ H(s) = \frac{0.5}{s+0.1-j3} + \frac{0.5}{s+0.1+j3}$$

$$\therefore\ H(z) = \frac{0.5}{1-e^{-(0.1-j3)T}\,z^{-1}} + \frac{0.5}{1-e^{-(0.1+j3)T}\,z^{-1}}$$

$$= \frac{0.5}{1-e^{-0.1}\,e^{j3}\,z^{-1}} + \frac{0.5}{1-e^{-0.1}\,e^{-j3}\,z^{-1}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

AJN notes

$$\therefore\ H(z) = \frac{0.5}{1 - e^{-(0.1 - j3)T} z^{-1}} + \frac{0.5}{1 - e^{-(0.1 + j3)T} z^{-1}}$$

$$= \frac{0.5}{1 - e^{-0.1} e^{j3} z^{-1}} + \frac{0.5}{1 - e^{-0.1} e^{-j3} z^{-1}}$$

$$= \frac{0.5(1 - e^{-0.1} e^{-j3} z^{-1}) + 0.5(1 - e^{-0.1} e^{j3} z^{-1})}{(1 - e^{-0.1} e^{j3} z^{-1})\,(1 - e^{-0.1} e^{-j3} z^{-1})}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

$$H(z) = \; = \frac{1 - 0.5\, e^{-0.1} z^{-1}(e^{j3} + e^{-j3})}{1 - e^{-0.1} z^{-1} (e^{j3} + e^{-j3}) + e^{-0.2} z^{-2}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

$$= \frac{1 - 0.5 \times (2\cos 3)\, e^{-0.1} z^{-1}}{1 - e^{-0.1} z^{-1} (2\cos 3) + e^{-0.2} z^{-2}}$$

$$= \frac{1 - (\cos 3)\, e^{-0.1} z^{-1}}{1 - 2(\cos 3)\, e^{-0.1} z^{-1} + e^{-0.2} z^{-2}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

$$= \frac{1 - 0.5 \times (2\cos 3)\, e^{-0.1} z^{-1}}{1 - e^{-0.1} z^{-1} (2\cos 3) + e^{-0.2} z^{-2}}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$= \frac{1 - (\cos 3)\, e^{-0.1} z^{-1}}{1 - 2(\cos 3)\, e^{-0.1} z^{-1} + e^{-0.2} z^{-2}}$$

Note : Evalutate $\cos\theta$ by keeping calculator in radian mode.

$$= \frac{1 + 0.8958 z^{-1}}{1 + 1.7916 z^{-1} + 0.8187 z^{-2}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

$$\frac{1}{s - p_i} \longrightarrow \frac{1}{1 - e^{p_i T} z^{-1}}$$

2. For multiples poles

$$\frac{1}{(s + p_i)^m} \longrightarrow \frac{(-1)^{m-1}}{(m-1)!} \frac{d^{m-1}}{dp_i^{m-1}} \left(\frac{1}{1 - e^{-p_i T} z^{-1}} \right)$$

3. For complex poles

$$\frac{s + a}{(s + a)^2 + b^2} \longrightarrow \frac{1 - e^{-aT} (\cos bT) z^{-1}}{1 - 2e^{-aT} (\cos bT) z^{-1} + e^{-2aT} z^{-2}}$$

$$\frac{b}{(s + a)^2 + b^2} \longrightarrow \frac{1 - e^{-aT} (\sin bT) z^{-1}}{1 - 2e^{-aT} (\cos bT) z^{-1} + e^{-2aT} z^{-2}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 2

Method - I

Given that, $H(s) = \dfrac{s+0.1}{(s+0.1)^2+9} = \dfrac{s+0.1}{(s+0.1)^2+3^2}$

For complex poles

$$\frac{s+a}{(s+a)^2+b^2} \longrightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

Analog to Digital filter Transformation:-Method 1 (IIT) ..solved example 1

AJN notes

Method - I

Given that, $H(s) = \frac{s+0.1}{(s+0.1)^2+9} = \frac{s+0.1}{(s+0.1)^2+3^2}$

Using transformation of equation (7.18) we can write,

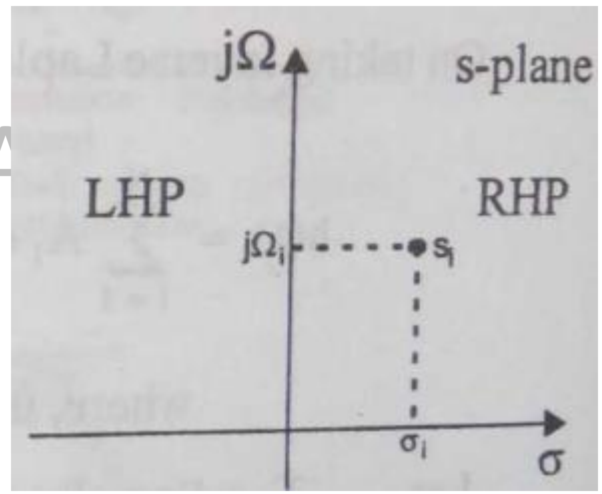
$$H(z) = \frac{1-e^{-0.1T}(\cos 3T)z^{-1}}{1-2e^{-0.1T}(\cos 3T)z^{-1}+e^{-2\times 0.1T}z^{-2}} = \frac{1-e^{-0.1}(\cos 3)z^{-1}}{1-2e^{-0.1}(\cos 3)z^{-1}+e^{-0.2}z^{-2}}$$

Put, T = 1

$$= \frac{1+0.8958z^{-1}}{1+1.7916z^{-1}+0.8187z^{-2}}$$

Alternatively,

$$H(z) = \frac{1+0.8958z^{-1}}{1+1.7916z^{-1}+0.8187z^{-2}} = \frac{1+0.8958z^{-1}}{z^{-2}(z^2+1.7916z+0.8187)} = \frac{z^2+0.8958z}{z^2+1.7916z+0.8187}$$


jΩ
s-plane
LHP
RHP
jΩi
si
σi
σ

The analog poles are given by the roots of the term $(s+p_i)$, for $i = 1, 2, 3, \ldots, N$. The digital poles are given by the roots of the term $(1-e^{-p_iT}z^{-1})$, for $i = 1, 2, 3, \ldots, N$. From equation (7.5) we can say that the analog pole at $s = -p_i$ is transformed into a digital pole at $z = e^{-p_iT}$

Consider the digital pole, $z_i = e^{-p_iT}$

Put, $-p_i = s_i$ in equation (7.7).

$$\therefore z_i = e^{-p_iT} = e^{s_iT}$$

We know that, "s_i" is a point on s-plane. Let the coordinates of s_i be σ_i and $j\Omega_i$ as shown in fig 7.3.

$$\therefore s_i = \sigma_i + j\Omega_i$$

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$$z_i = e^{(\sigma_i + j\Omega_i)T} = e^{\sigma_i T} e^{j\Omega_i T}$$

We know that "z_i" is a complex number. Hence "z_i" can be expressed in polar coordinates as, $z_i = |z_i| \angle z_i$

$$\therefore |z_i| \angle z_i = e^{\sigma_i T} e^{j\Omega_i T} \quad \text{..... (7.10)}$$

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$$|z_i| = e^{\sigma_i T} \text{ and } \angle z_i = \Omega_i T \qquad \text{.....(7.11)}$$

From equation (7.11) the following observations can be made.

1. If $\sigma_i < 0$ (i.e., σ_i is negative), then the analog pole "s_i" lie on Left Half (LHP) of s-plane. In this case, $|z_i| < 1$, hence the corresponding digital pole "z_i" will lie inside the unit circle in z-plane.

2. If $\sigma_i = 0$ (i.e., real part is zero), then the analog pole "s_i" lie on imaginary axis of s-plane. In this case, $|z_i| = 1$, hence the corresponding digital pole "z_i" will lie on the unit circle in z-plane.

N A J N N O t e s

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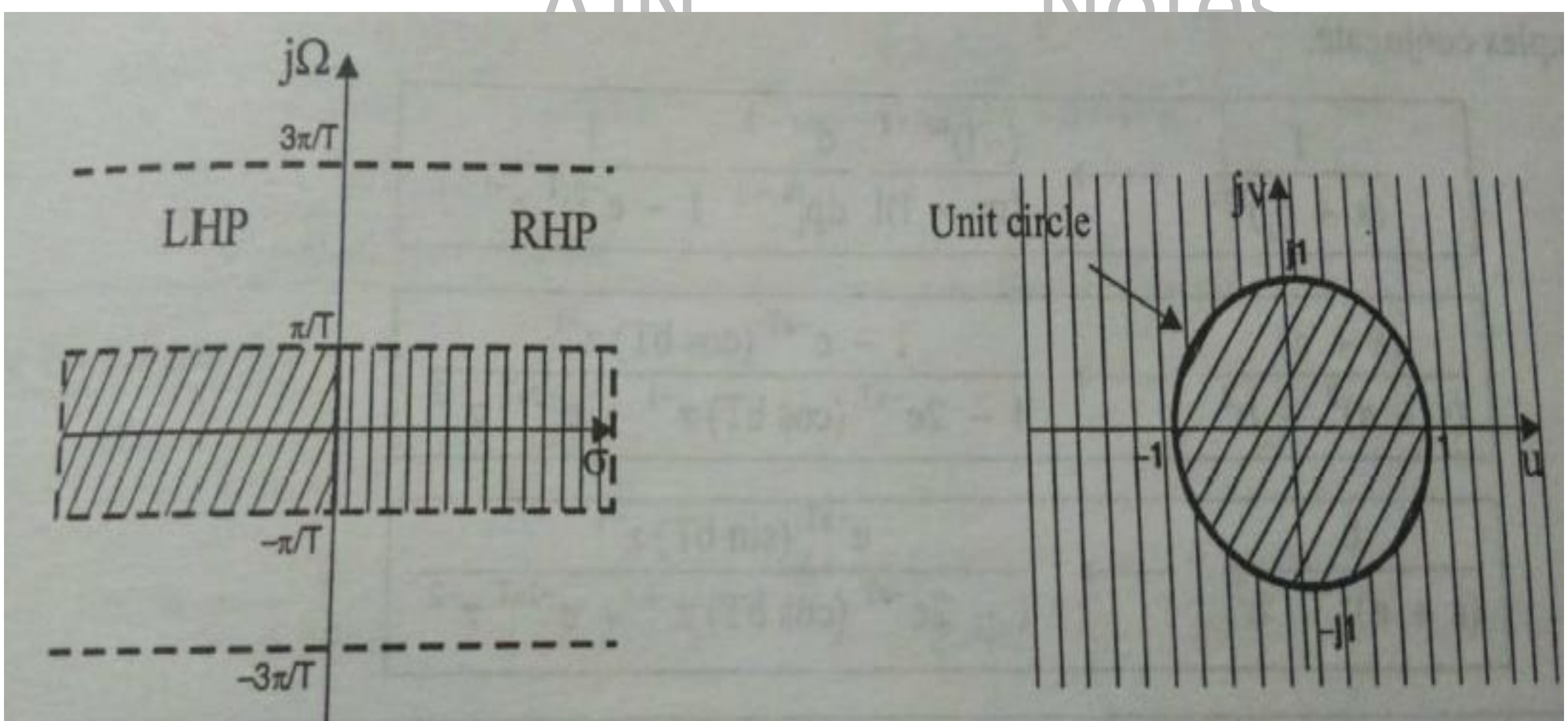
AJN notes

3. If $\sigma_i > 0$ (i.e., σ_i is positive), then the analog pole "s_i" lie on the Right Half (RHP) of s-plane. In this case $|z_i| > 1$, hence the corresponding digital pole will lie outside the unit circle in z-plane.



Aliasing problem in IIT

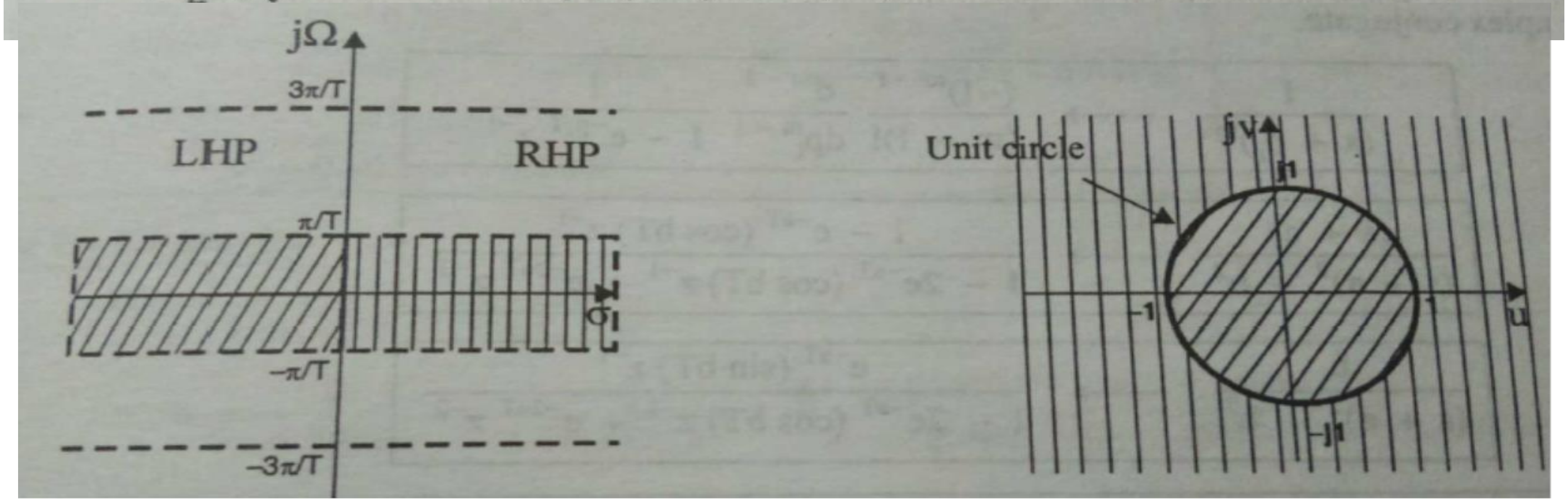
The above discussions are applicable for mapping any point on s-plane to z-plane. In general the impulse invariant transformation maps all points in the s-plane given by,

$$s_i = \sigma_i + j\Omega_i + j\frac{2\pi k}{T}, \quad \text{for } k = 0, \pm 1, \pm 2 \ldots\ldots \qquad \ldots\ldots (7.12)$$

into a single point in the z-plane as

$$z = e^{\left(\sigma_i + j\Omega_i + \frac{j2\pi k}{T}\right)T} = e^{\sigma_i T}\, e^{j\Omega_i T}\, e^{j2\pi k} = e^{\sigma_i T}\, e^{j\Omega_i T} \qquad \ldots\ldots (7.13)$$

For integer k, $e^{j2\pi k} = 1$



Aliasing problem in IIT

From equations (7.12) and (7.13) we can say that the strip of width $2\pi/T$ in the s-plane for values of s in the range $-\pi/T \le \Omega \le +\pi/T$ is mapped into the entire z-plane. Similarly the strip of width $2\pi/T$ in the s-plane for values of s in the range $\pi/T \le \Omega \le 3\pi/T$ is also mapped into the entire z-plane. Likewise the strip of width $2\pi/T$ in the s-plane for values of s in the range $-3\pi/T \le \Omega \le -\pi/T$ is also mapped into the entire z-plane.

In general any strip of width $2\pi/T$ in the s-plane for values of s in the range, $(2k-1)\pi/T \le \Omega \le (2k+1)\pi/T$ (where k is an integer), is mapped into the entire z-plane. The left half portion of each strip in s-plane maps into the interior of the unit circle in z-plane, right half portion of each strip in s-plane maps into the exterior of the unit circle in z-plane and the imaginary axis of each strip in s-plane maps into the unit circle in z-plane as shown in fig 7.4. Therefore we can say that the impulse invariant mapping is many-to-one mapping (and does not provide one-to-one mapping).

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Aliasing problem in IIT

AJN notes

The above discussions are applicable for mapping any point on s-plane to z-plane. In general the impulse invariant transformation maps all points in the s-plane given by,

$$s_i = \sigma_i + j\Omega_i + j\frac{2\pi k}{T}, \quad \text{for } k = 0, \pm 1, \pm 2 \ldots\ldots \qquad \ldots\ldots (7.12)$$

into a single point in the z-plane as

For integer k, $e^{j2\pi k} = 1$

$$z_i = e^{\left(\sigma_i + j\Omega_i + \frac{j2\pi k}{T}\right)T} = e^{\sigma_i T}\, e^{j\Omega_i T}\, e^{j2\pi k} = e^{\sigma_i T}\, e^{j\Omega_i T} \qquad \ldots\ldots (7.13)$$

From equations (7.12) and (7.13) we can say that the strip of width $2\pi/T$ in the s-plane for values of s in the range $-\pi/T \le \Omega \le +\pi/T$ is mapped into the entire z-plane. Similarly the strip of width $2\pi/T$ in the s-plane for values of s in the range $\pi/T \le \Omega \le 3\pi/T$ is also mapped into the entire z-plane. Likewise the strip of width $2\pi/T$ in the s-plane for values of s in the range $-3\pi/T \le \Omega \le -\pi/T$ is also mapped into the entire z-plane.

In general any strip of width $2\pi/T$ in the s-plane for values of s in the range, $(2k-1)\pi/T \le \Omega \le (2k+1)\pi/T$ (where k is an integer), is mapped into the entire z-plane. The left half portion of each strip in s-plane maps into the interior of the unit circle in z-plane, right half portion of each strip in s-plane maps into the exterior of the unit circle in z-plane and the imaginary axis of each strip in s-plane maps into the unit circle in z-plane as shown in fig 7.4. Therefore we can say that the impulse invariant mapping is many-to-one mapping (and does not provide one-to-one mapping).

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